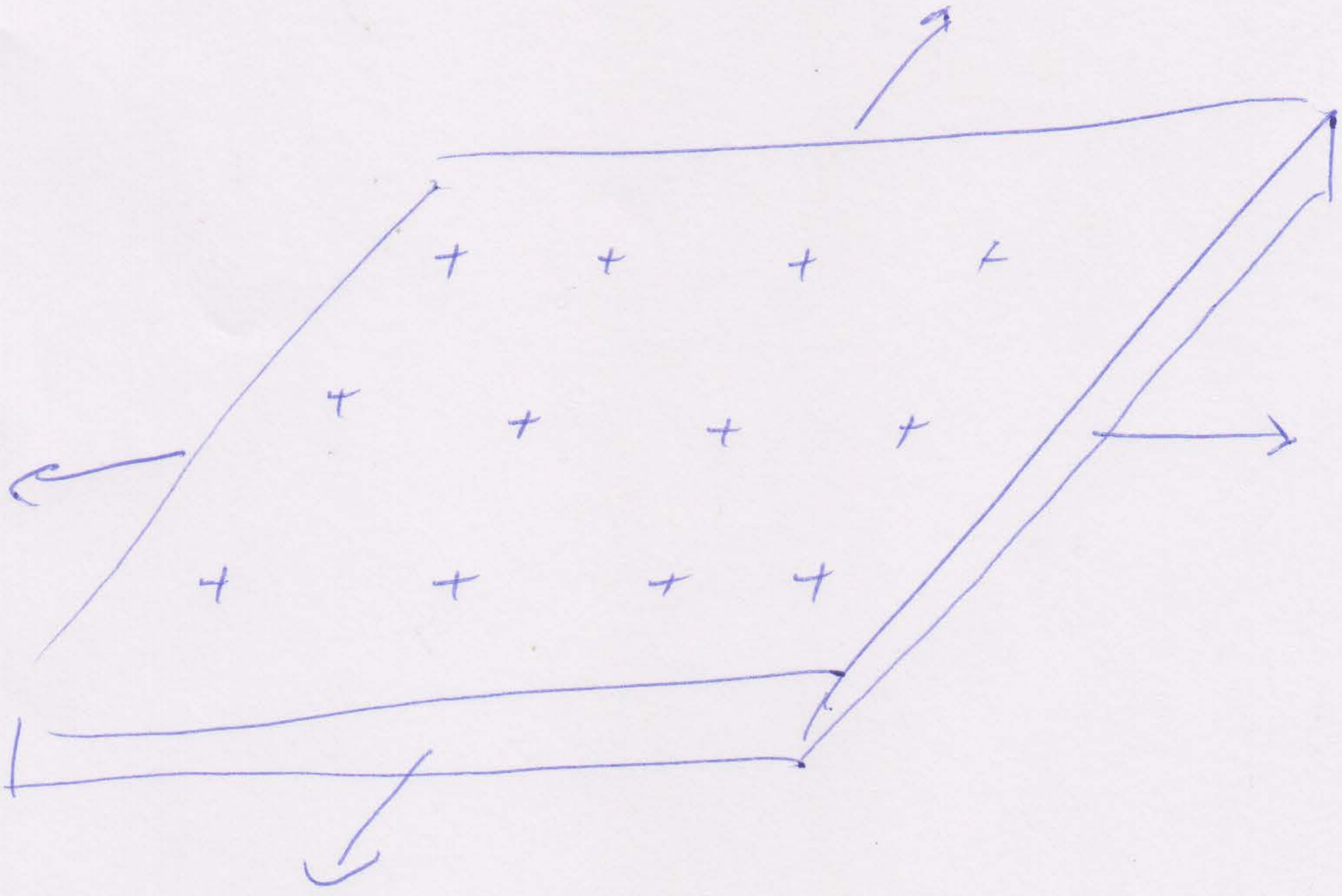
Find the electric field a distance z away from an infinitely-large & uniformly-charged sheet of charge.

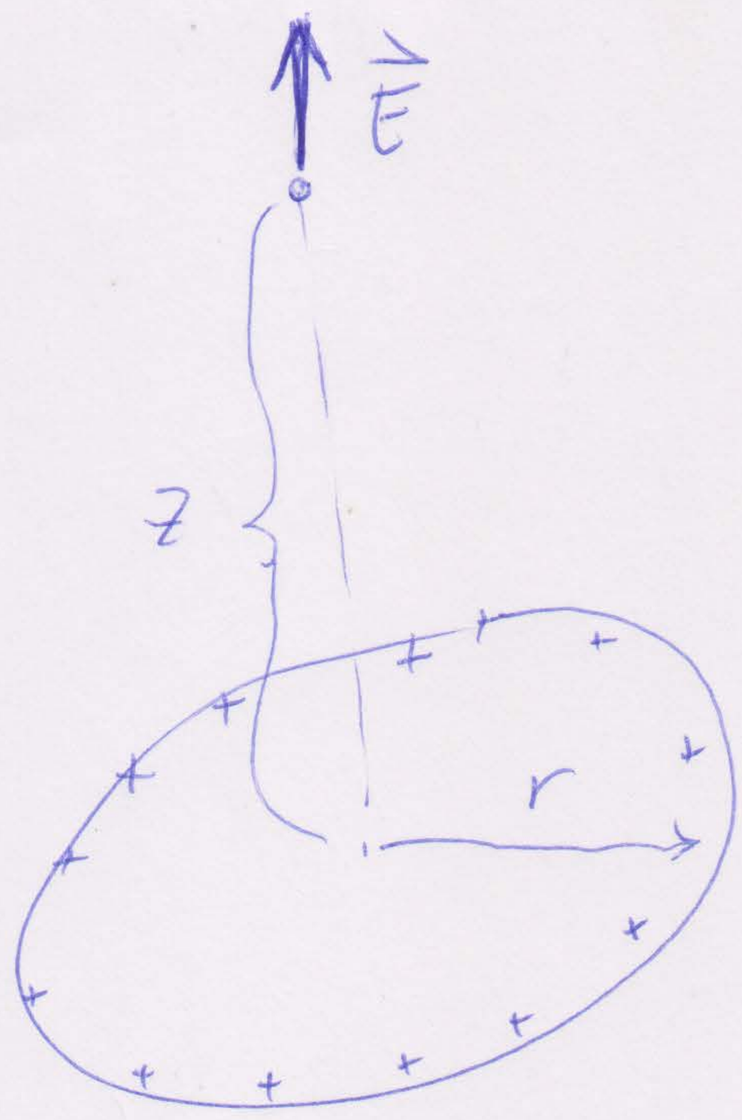


Assume the sheet has a surface charge density (charge per unit area) σ.

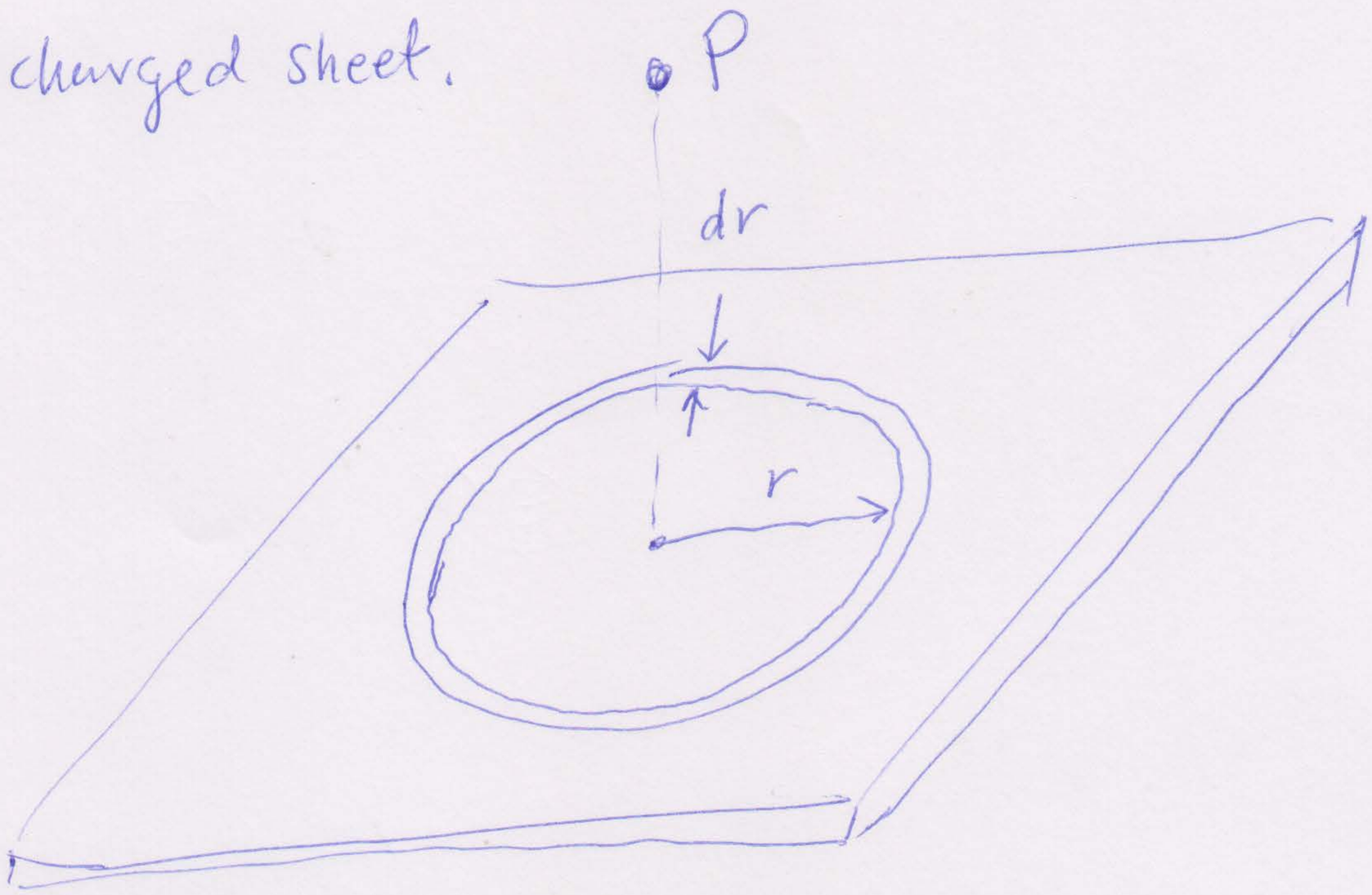
Strategy: Consider forming a sheet of charge by summing a bunch of charged rings of different sizes.

Already know that the electric field a height z above the centre of a charged ring is given by

$$|\vec{E}| = \frac{2\pi r k_e \lambda z}{(z^2+r^2)^{3/2}}$$



A ring of radius r & thickness dr, placed within the charged sheet.



The ring has area $\underbrace{2\pi r}_{\text{circumference}}\,\underbrace{dr}_{\text{thickness}}$

$\therefore$ Its charge is $Q_{ring} = \sigma(2\pi r\, dr)$

$\therefore$ charge per unit length of the ring is

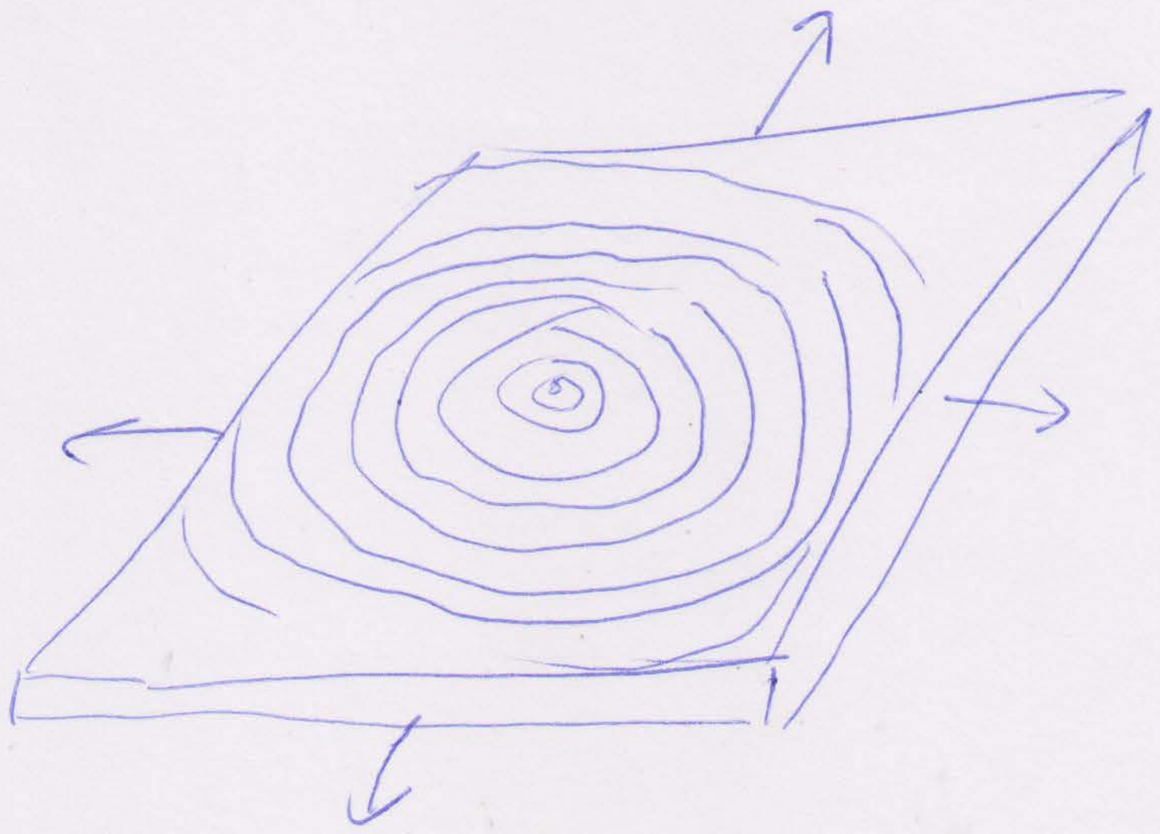
$$\lambda = \frac{Q_{ring}}{2\pi r} = \sigma\, dr$$

$\therefore$ the electric field at P due to this one ring is

$$dE = \frac{2\pi r\, k_e \sigma z\, dr}{(z^2 + r^2)^{3/2}}$$

To get the total electric field due to the sheet, sum contributions from all rings

$$\vec{E}_{sheet} = \sum_{\text{all rings}} d\vec{E} = 2\pi k_e \sigma \sum_{\text{all rings}} \frac{r z\, dr}{(z^2+r^2)^{3/2}} \quad (1)$$



That is, consider rings starting from $r = 0$ to $r = \infty$.

The sum given in Eq. ① can be expressed as the following integral:

$$E_{sheet} = 2\pi k_e \sigma \int_{r=0}^{\infty} \frac{r z \, dr}{(z^2+r^2)^{3/2}}$$

This integral can be solved using the substitution

$$u = z^2 + r^2$$

$$du = 2r\,dr \Rightarrow r\,dr = \frac{du}{2}$$

when $r = 0$, $u = z^2$

$r = \infty$, $u = \infty$

$$E_{sheet} = 2\pi k_e \sigma \int_{u=z^2}^{\infty} \frac{z\,du}{2(u)^{3/2}}$$

$$= \frac{2\pi k_e \sigma z}{2} (-2) \frac{1}{u^{1/2}} \Bigg|_{u=z^2}^{\infty}$$

$$\therefore E_{sheet} = -2\pi k_e \sigma z \left(\frac{1}{\sqrt{\infty}} - \frac{1}{\sqrt{z^2}} \right)$$

$$= 2\pi k_e \sigma$$

replace k_e w/ $k_e = \frac{1}{4\pi\varepsilon_0}$

$$E_{sheet} = \frac{2\pi\sigma}{4\pi\varepsilon_0} \quad \therefore \boxed{E_{sheet} = \frac{\sigma}{2\varepsilon_0}}$$

The electric field due to a uniformly-charged sheet is independent of the distance z away from the sheet.